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Donor metastable states and resonant electron–phonon interaction in n-GaAs

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Abstract. Far-infrared photoconductivity measurements on n-GaAs using a far-infrared laser and a Fourier-transform spectrometer are reported. They extend previous measurements to higher magnetic fields (14 T) and higher laser energies (330 cm^{-1}) and, in particular, give results in those regions of the spectra where resonant polaron effects on transitions to metastable states are clearly present. To describe the field dependence of the transition energies, a new variational-type approach for the calculation of the energies of the metastable states has been developed. As such an approach yields analytical expressions for the wave functions, the effects of resonant polaron interactions on the transition energies can be calculated. Experimental and theoretical transition energies are compared for the more important transitions to metastable states, including resonant polaron effects, and very good agreement is obtained even though the theoretical model is very simple.

1. Introduction

In the last decade, many papers have been published which describe the far-infrared spectrum of donor impurities in GaAs in the presence of high magnetic fields. Originally, the simple donor D^0 was studied in bulk materials (we cite Cheng *et al* (1993b) and Shi *et al* (1993, 1994a) as two examples of recent work) but recently the ionized donor D^- has been extensively studied also (e.g. Shi *et al* 1994b, Peeters *et al* 1993, Dzuybenko 1992, Dzyubenko and Sivachenko 1993, Dunn *et al* 1993). In addition, there is much current interest in donors in quantum-well (QW) and superlattice systems. However, we limit our discussion here to the simple D^0 donor in bulk n-GaAs because such systems form the basis of studies upon which the other more complicated systems build.

A number of different experimental techniques have been used in these studies on a variety of samples. Most of the many publications on this subject concentrate on the hydrogen-like transitions such as $1s_0$ to $2p_\pm$ (e.g. Stillman *et al* 1969, Low *et al* 1982, Allan *et al* 1985). However, the spectrum contains many additional lines at higher energies.

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Although these lines are weaker than the $1s_0$-to-$2p_\pm$ transitions, they are well defined and may be detected easily by conventional far-infrared photoconductivity (FIRPC) experiments. In view of the importance of impurities in the performance of device material such as GaAs, it is necessary to have an accurate identification of all transitions observed. It was recognized some time ago (for example, Golubev *et al* 1988, Wagner and Prettl 1988, Armistead *et al* 1989, van Klarenbosch *et al* 1990a, Burghoorn *et al* 1990) that much of the fine structure observed, particularly at the higher energies, arose from transitions to the so-called metastable or auto-ionizing states.

The purpose of our work on this subject is to give a detailed description of the shallow donor spectrum in GaAs, with the emphasis on the transitions to the metastable states. Moreover, the influence of the electron–phonon interaction will be considered as the magnitude of that interaction gives valuable information on the metastable state wave functions. Our previous experiments on a high-purity n-GaAs sample using a far-infrared laser (van Klarenbosch *et al* 1990b) up to 264 cm^{-1} and magnetic fields up to 6 T have been extended to 14 T. In addition, measurements have been performed up to 330 cm^{-1} (that is, above the longitudinal optical (LO) phonon energy of 296 cm^{-1}) using a Fourier-transform spectrometer, to study the resonant electron–phonon interaction of some of the metastable states. The other most recent work which is closest to this is that described by Cheng *et al* (1993a); however their work on bulk GaAs considers transitions to hydrogen-like 2p states only.

In order to give a detailed explanation of the results, theoretical work has been undertaken in which the energies of transitions to the metastable states have been calculated. For that purpose, we have two existing models available. The first, which is a modification of that described by Simola and Virtamo (1978), was used by us recently (Hawksworth *et al* 1992). Although this can give the energies of *all* metastable states, the method does not yield explicit analytical forms for the wave functions. This makes it very difficult to calculate accurately the electron–phonon interaction. A second and novel approach to the calculation of metastable states in quantum-well systems has been given very recently by two of us and co-workers (Kuhn *et al* 1994). Whereas this method could very easily be converted to calculations for bulk GaAs, it is also numerical and thus no explicit analytical form for the wave function can be extracted.

A new approach, involving a simple variational procedure, will be developed in this paper for the first time and the results obtained compared to the experimental results on some strong transitions to metastable states. This method does yield analytical expressions for the wave functions, so special emphasis will be placed on the resonant electron–phonon (polaron) interactions (Devreese 1972). In this paper, the method used is in its simplest form and has only been developed for some of the metastable states of interest. Fortunately however, these are the states involved in the most prominent transitions in the spectrum. In a second paper, we intend to present further experimental and theoretical details of the spectrum at relatively low energies (i.e. away from the regions where the resonant electron–phonon interaction is strong). This will include refinements in the theory and comparisons with other calculations including those of the method given by Hawksworth *et al* (1992). Also, other weaker transitions and those involving transitions within excited states will be discussed in the second paper.

It is useful to have a convenient notation to describe all of the states of the donor impurity. We refer to both the hydrogenic and metastable states using the high-field notation (N, m, ν), where N is the principal Landau quantum number, m is the usual magnetic quantum number and ν is the number of nodes of the wave function in the z-direction (van Klarenbosch *et al* 1990a). Thus the ground 1s-like state is written as (0, 0, 0), $2p_+$ as

$(1, 1, 0)$ and $2p_-$ as $(0, -1, 0)$, for example. In the Faraday configuration ($\boldsymbol{E} \perp \boldsymbol{B}$) used in all experiments, the electric-dipole selection rules show that transitions from the ground state to states $(N, \pm 1, \nu)$ for ν even are allowed (although other transitions may be allowed due to perturbative mixings). The strongest transitions observed are expected to be to states with $\nu = 0$.

2. Experimental measurements

The samples used in this work are 10 μm thick epitaxial layers of n-GaAs on a semi-insulating substrate, intentionally doped with Si ($N_d = 5 \times 10^{14}$ cm^{-3} and $N_a = 1 \times 10^{14}$ cm^{-3}). They were grown by MBE at the Philips Research Laboratory, Redhill, UK. (The impurity concentrations were determined originally at source and confirmed by temperature-dependent Hall measurements. The latter gave information on the acceptor/donor compensation ratio. From studies of the observed spectra, the concentration of donors other than Si is estimated to be not greater than about 5% of the Si concentration.)

Our previous magneto-optical work on this system was performed using a conventional optically pumped FIR laser for laser energies up to 264 cm^{-1} and a 6 T superconducting magnet (van Klarenbosch *et al* 1988, 1990a, b). Those experiments were extended by FIR laser spectroscopy in fields up to 14 T at the High-Field Magnet Laboratory of the University of Nijmegen. The results for some of the transitions from both sets of experiments are shown together in figures 1–3.

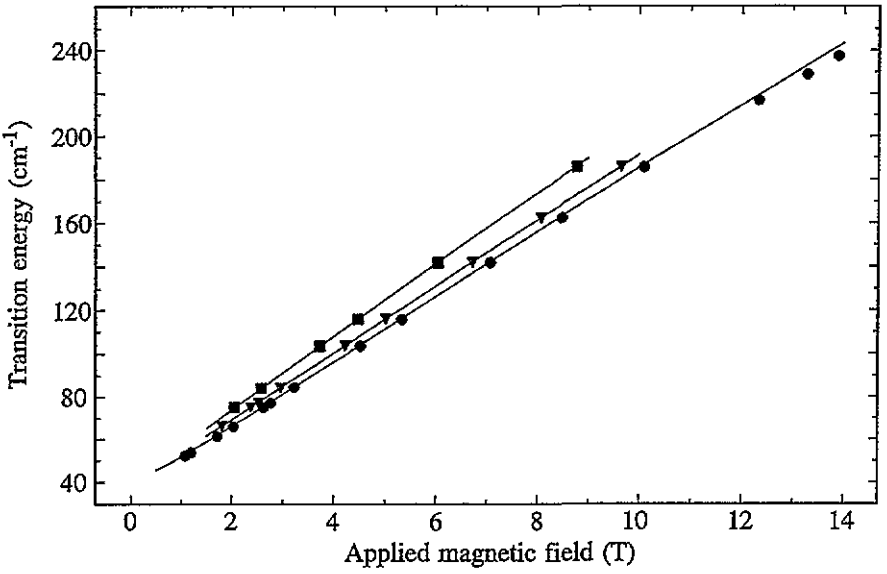

Figure 1. Calculated transition energies as a function of magnetic field for the $1s_0$ to (N, m, ν) metastable states with $N = 1$. Included in the calculations are corrections for conduction band non-parabolicity and (zero-field) polaron effect. The experimental results are shown as follows: ● for state $(1, -1, 0)$, ▼ for $(1, -2, 0)$ and ■ for $(1, 0, 1)$.

For energies below that of the transverse optical (TO) phonon at 270 cm^{-1}, where the electron–phonon interaction is small, a nearly linear field–energy relation for the transitions

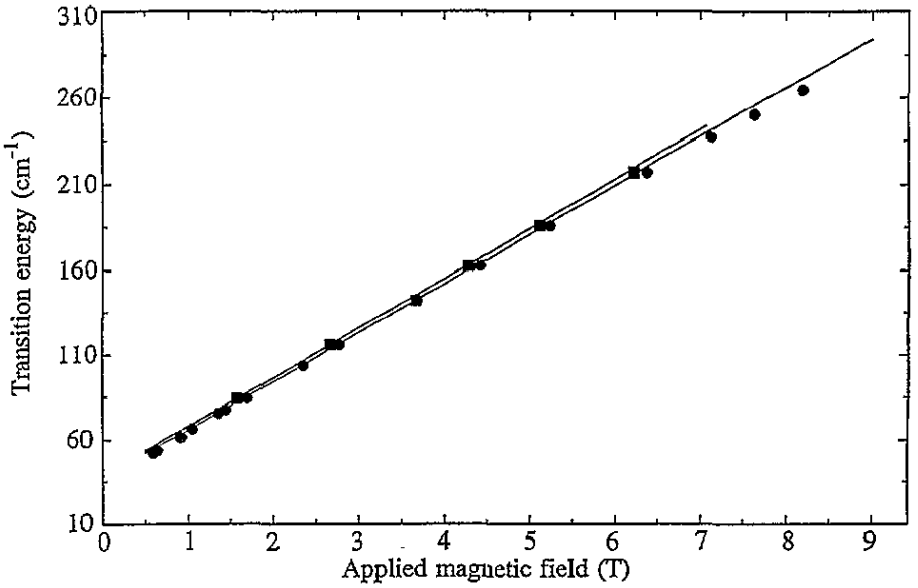

Figure 2. As figure 1 but with $N = 2$. The experimental results are shown as follows: ● for (2, 1, 0) and ■ for (2, −1, 0).

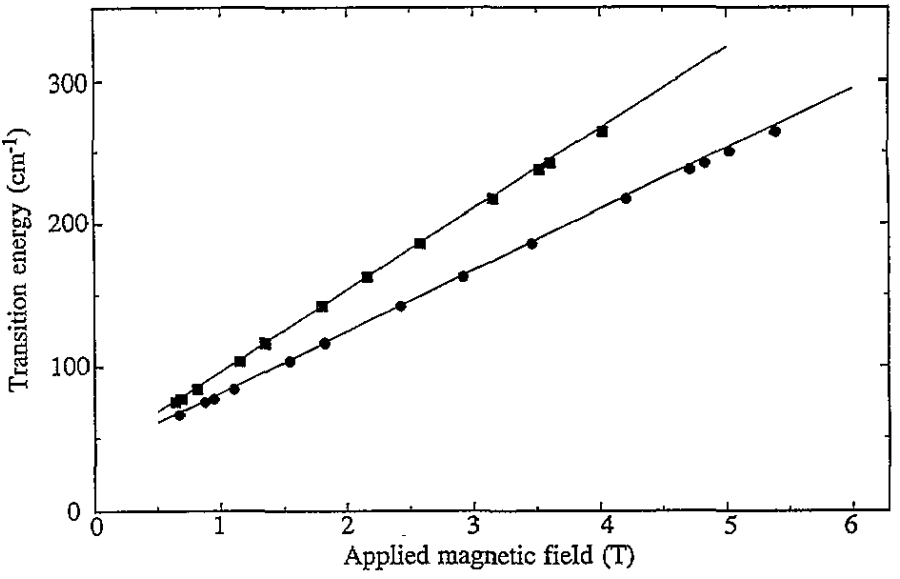

Figure 3. As figure 1 but with $N = 3$ and 4. The experimental results are shown as follows: ● for (3, 1, 0) and ■ for (4, 1, 0).

is observed. In that energy region therefore experiments at a relatively small number of different laser wavelengths are sufficient to determine accurately the field-dependent transition energies. For energies near that of the LO-phonon energy, however, this relation becomes very non-linear due to the resonant electron–phonon interaction. The limited

number of available FIR laser lines in that energy region ($\geqslant 295$ cm^{-1}) therefore cannot give the information necessary for the detailed observation of the resonant polaron effect. Consequently in that energy region FIRPC experiments have been performed using a BOMEM DA8 FIR Fourier-transform spectrometer. The light from the spectrometer is guided to the sample, mounted in the centre of a 9 T superconducting magnet, using a telescope, waveguide and a condensing cone. Because of the Restrahlen reflection band of GaAs, no spectra could be observed between 270 and 296 cm^{-1}. Also, no signals were seen above 330 cm^{-1} probably due to system limitations. In figure 4, the Fourier-transform data on the transitions to the (3, 1, 0) and (4, 1, 0) metastable states are given; a clear resonant behaviour is observed in the 296–330 cm^{-1} region resulting from the interaction with the nearby $1s_0$ state with one LO-phonon excitation and the $2p_-$ state with one LO-phonon excitation.

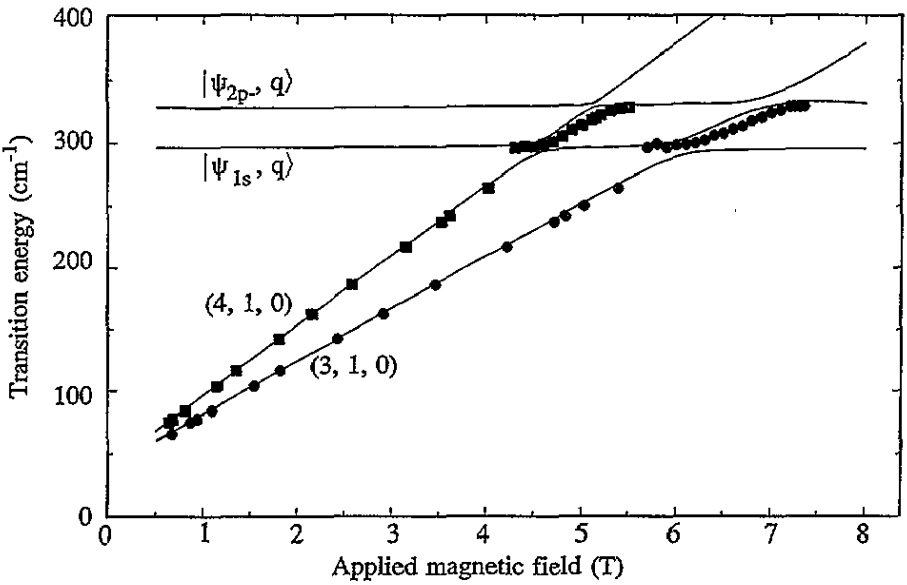

Figure 4. Calculated transition energies of the $1s_0$ to the (3, 1, 0) and (4, 1, 0) metastable states including the full effects of polaron interaction and band non-parabolicity for the complete field range shown. The resonant polaron effect is clearly seen to lead to avoided crossings of the (3, 1, 0) and (4, 1, 0) metastable states with the states $|\Psi_{1s}, q\rangle$ and $|\Psi_{2p-}, q\rangle$. The experimental points are also shown by ■ and ●.

3. The theory of metastable states

3.1. The basic equations

We begin with the non-relativistic Hamiltonian used by Simola and Virtamo (1978). This treats the impurity like a hydrogen atom with an electron of effective mass m, placed at the origin and in a uniform magnetic field $\boldsymbol{B}$. Neglecting spin, the Hamiltonian is

$$\mathcal{H} = \frac{\pi^2}{2m} + V(r) \tag{3.1}$$

where $\pi = p + eA$ is the momentum operator, A is the vector potential of the magnetic field and $V(r)$ is the Coulomb potential. The z-axis is chosen to be along B.

In the absence of the Coulomb term, the eigenstates of (3.1) are the well known Landau functions $\Phi_{Nm}(\rho, \phi)$ which, in cylindrical coordinates, have the form

$$\Phi_{Nm}(\rho, \phi) = \frac{1}{\sqrt{2\pi\lambda^2}} e^{im\phi} e^{-(1/2)\zeta} \zeta^{(1/2)|m|} P_{Nm}(\zeta) \tag{3.2}$$

where N and m are the Landau and magnetic quantum numbers respectively, ζ $(= \rho^2/(2\lambda^2))$ is a dimensionless variable and λ $(= \sqrt{(\hbar/eB)})$ is the magnetic length. The polynomials P_{Nm} are closely related to the associated Laguerre polynomials such that

$$P_{Nm}(\zeta) = \frac{1}{\sqrt{N!s!}} \sum_{k=0}^{\min(N,s)} \frac{(-1)^k}{k!} \frac{N!}{(N-k)!} \frac{s!}{(s-k)!} \zeta^{\min(N,s)-k} \tag{3.3}$$

where $s \equiv N - m$. It is useful to introduce operators

$$\pi_{\pm} = \frac{\lambda}{\hbar} \frac{(\pi_x \pm i\pi_y)}{\sqrt{2}} \tag{3.4}$$

which act as raising and lowering operators on N and m such that

$$\begin{aligned} \pi_+ \Phi_{Nm}(\rho, \phi) &= \sqrt{N+1}\, \Phi_{(N+1)(m+1)}(\rho, \phi) \\ \pi_- \Phi_{Nm}(\rho, \phi) &= \sqrt{N}\, \Phi_{(N-1)(m-1)}(\rho, \phi). \end{aligned} \tag{3.5}$$

When the Coulomb term is included, these definitions can still be used such that, when the problem is formulated in dimensionless form, the Schrödinger equation becomes

$$\left[4\beta\left(\pi_+\pi_- + \frac{1}{2}\right) - \frac{d^2}{dz^2} - \frac{2}{r}\right] \Psi_{Nmv}(r) = E_{Nmv} \Psi_{Nmv}(r) \tag{3.6}$$

where $\Psi_{Nmv}(r)$ is the total wave function and E_{Nmv} is the energy. In (3.6), the unit of length is the effective Bohr radius a_0 $(= 4\pi\epsilon\hbar^2/(me^2))$ and the unit of energy is the effective Rydberg R $(= e^2/(8\pi\epsilon a_0))$. β $(= (e\hbar B)/(4mR))$ is a dimensionless measure of magnetic field.

In the adiabatic limit of very strong magnetic fields, when the cylindrically symmetric magnetic field dominates, the wave function $\Psi(r)$ for the hydrogenic impurity can, to a good approximation, be written in the separated form

$$\Psi_{Nmv}(r) = \Phi_{Nm}(\rho, \phi) f^N_{mv}(z) \tag{3.7}$$

where $f^N_{mv}(z)$ is a function of z only.

One method of solution for f^N_{mv} is to substitute (3.7) into (3.6) and integrate over ρ and ϕ to give the differential equation

$$\left[-\frac{d^2}{dz^2} + 2\beta(2N+1) - I_r^{NN'}\right] f^N_{mv}(z) = E_{Nmv} f^N_{mv}(z) \tag{3.8}$$

where

$$I_r^{NN'} = 2\beta^{1/2} \int_0^{\infty} \frac{P_N^m(\zeta) P_{N'}^m(\zeta) e^{-\zeta} \zeta^{|m|}}{\sqrt{\zeta + (\beta^{1/2} z)^2}}\, d\zeta. \tag{3.9}$$

In principle, this equation can be solved for a given number of nodes ν using standard integration routines. This was the approach used by Simola and Virtamo (1978). (Note that the unit of energy used by Simola and Virtamo (1978) is equal to $2R$ with R defined above.) However, the solutions $\Psi_{Nm\nu}$ are rather poor for the region of relatively low magnetic field accessible experimentally, as strictly they apply in the adiabatic limit only. For weaker fields, it is therefore necessary to take into account the coupling between Landau levels due to the spherically symmetric Coulomb potential. This gives a system of coupled differential equations for the $f^N_{m\nu}$ which can be solved numerically using an iterative procedure (Simola and Virtamo 1978) although this is a rather lengthy process.

It is also possible to obtain an asymptotic relation between the functions $f^N_{m\nu}$ for large but non-infinite fields which results in an eigenvalue problem which can be solved in the same manner as that for the adiabatic case (Simola and Virtamo 1978). This method was used by us in the results given by Hawksworth *et al* (1992) following our earlier work described by Pearl et al (1992). However, many technical problems have been encountered in inserting appropriate boundary conditions for numerical solutions of the equations. Also, our subsequent calculations require explicit analytical forms for $f^N_{m\nu}(z)$ in order for us to incorporate polaron corrections. Therefore, in this paper we develop an alternative procedure for solving the equation (3.8) based on a variational-type approach.

3.2. A variational calculation

The aim is to obtain values for $E_{Nm\nu}$ and $f^N_{m\nu}(z)$ using simple variational methods. To do this we treat $f^N_{m\nu}(z)$ as a trial function of the form

$$f^N_{m\nu}(z) = z^\nu e^{-b_{Nm}z^2} \tag{3.10}$$

for the two cases where $\nu = 0$ and 1 and where b_{Nm} is the variational parameter. For these particular cases, ν gives the parity of the wave function in the z-direction. The energy of the metastable state and its associated wave function are then found by minimizing the energy expression

$$E_{Nm\nu} = 2\beta(2N+1) + 3^\nu b_{Nm} - (4b_{Nm})^\nu \sqrt{\frac{8b_{Nm}}{\pi}} \int_0^\infty I_r^{NN} z^{2\nu} e^{-2b_{Nm}z^2}\, dz \tag{3.11}$$

with respect to the parameter b_{Nm}. The resultant normalized wave function is given by

$$\Psi^N_{mq}(r) = \frac{a_0}{\sqrt{2\pi\lambda^2 I^{Nm\nu}_{Z0}}} e^{im\phi} e^{-(1/2)\zeta} \zeta^{(1/2)|m|} P_{Nm}(\zeta) z^\nu e^{-b_{Nm}z^2} \tag{3.12}$$

where $I^{Nm\nu}_{Z0}$ is the integral

$$I^{Nm\nu}_{Z0} = \int_{-\infty}^{\infty} z^{2\nu} e^{-2b_{Nm}z^2}\, dz = \frac{1}{(4b_{Nm})^\nu} \sqrt{\frac{\pi}{2b_{Nm}}}. \tag{3.13}$$

4. Results

4.1. The calculation of the transition energies

In order to calculate the transition energies corresponding to the FIR spectrum, the energies of both the ground and relevant excited states are required. However, as the electron is most tightly bound to the nucleus when it is in the lowest-energy states, the above method of calculation is not sufficiently accurate for the $1s_0$ and $2p_-$ states. Therefore, the method of Dunn and Pearl (1991) has been used for these states in which the hydrogenic wave functions are written in the form

$$\Psi_H = \sum_i c_i \rho^{|m|} e^{im\phi} e^{-(\beta_i+\delta)\rho^2} z^q e^{-\alpha_i z^2}. \tag{4.1}$$

c_i are the wave function coefficients, α_i and β_i are numbers chosen from the Gaussian expansion of the Slater-like function of the hydrogenic wave function and $\delta = 0.2\beta$ (to allow for the constriction of the wave function in the ρ direction with increased magnetic field).

The transition energies from the $1s_0$ ground state to various excited metastable states have been calculated. However, to compare these with the experimental data it is necessary to take into account effects such as band non-parabolicity and electron–phonon interactions (the polaron effect) on the energies of both the ground and excited metastable states. Cheng *et al* (1993b) correct for band non-parabolicity and polaron effects in their study of hydrogenic states in bulk GaAs. Here we have taken into account only the lowest-order correction for the electron–phonon interaction by incorporating the factor $(1 + \alpha/6)$ implicitly in the value taken for the effective mass of the electron. This implies that, for GaAs, $a_0 = 100.06$ Å, $R = 46.11$ cm^{-1} and $\beta = 0.076B$ (for B in tesla) as $\alpha_0 = 0.068$. For all transitions below the Reststrahlen band, the additional field- (i.e. energy-) dependent electron–phonon interaction has been neglected. However, we have considered the effects of non-parabolicity of the conduction band. To do this we use the standard Kane model for the corrected energy E and write (Peeters *et al* 1992)

$$E = E^0\left(1 - \delta'\frac{E^0}{E_g}\right) \tag{4.2}$$

where E^0 is the energy calculated for a parabolic conduction band; E_g is the GaAs band gap (= 1520 meV). The parameter δ' is taken to be 0.73 (Vrehen 1968).

The corrected calculated transition energies, together with the corresponding experimental data (which had been identified prior to these theoretical calculations), are given in figures 1–3. As can be seen from the figures, the simple model for the metastable impurity states is in good agreement with the available experimental data although this agreement becomes less satisfactory with increasing energies due to the proximity of resonant polaron effects.

One disadvantage of this variational method is that, so far, the function $f^N_{m\nu}(z)$ has been limited to those functions for which $\nu = 1$ and 0. A more sophisticated form of $f^N_{m\nu}(z)$ would be required for states with larger values of ν; this is a problem to solve in the future. Also for possible future consideration is the need to consider mixing between the Landau levels such as that summarized in section 3.1.

4.2. The effects of resonant polaron interactions

The correction ΔE_g to the energies of the metastable state Ψ_g from the polaron effect is given in second-order perturbation theory by (Shi *et al* 1991)

$$\Delta E_g = -\sum_h \sum_q \frac{|\langle h, q|\mathcal{H}_F|g, 0\rangle|^2}{E_h + \hbar\omega_{LO} - E_g - \Delta E_g} \tag{4.3}$$

where $|g, 0\rangle$ is the wave function of the unperturbed state of energy E_g with no LO phonons, and $|h, q\rangle$ is the wave function of an unperturbed electronic state of energy E_h with an LO phonon of wave vector q with energy $\hbar\omega_q \simeq \hbar\omega_{LO} = 296\ \text{cm}^{-1}$. $\mathcal{H}_F$ is the Fröhlich polaron Hamiltonian given by

$$\mathcal{H}_F = \sum_q (V_q a_q e^{iq\cdot r} + V_q^* a_q^\dagger e^{-iq\cdot r}). \tag{4.4}$$

V_q and V_q^* are constants for a given q, where

$$|V_q|^2 = \frac{4\pi\alpha}{V}\sqrt{\frac{\hbar}{2m\omega_{LO}}}\left(\frac{\hbar\omega_{LO}}{q}\right)^2. \tag{4.5}$$

a_q and $a_q^\dagger$ are phonon annihilation and creation operators respectively and V is the volume of the system under investigation.

The region of particular interest is where the $|\Psi_{310}, 0\rangle$, $|\Psi_{410}, 0\rangle$ metastable states have energies very close to the $|\Psi_{1s_0}, q\rangle$ and $|\Psi_{2p_{-1}}, q\rangle$ hydrogen-like states. We calculate the polaron interaction between these states using the form of wave function (3.10) from the variational procedure for the metastable states and the hydrogenic form (4.1) from Dunn and Pearl (1991) for the hydrogen-like states. Thus we have, for example,

$$\sum_q |\langle \Psi_{1s_0}, q|\mathcal{H}_F|\Psi_{310}, 0\rangle|^2$$

$$= \frac{3\pi^3\alpha}{4} R^{1/2}(\hbar\omega_{LO})^{3/2}\frac{\beta^2}{\pi I_{Z0}^{310}} \sum_j \sum_i \frac{c_i^{1s_0} c_j^{1s_0}}{\sqrt{(\alpha_i + b_{Nm})(\alpha_j + b_{Nm})D_i^2 D_j^2}}$$

$$\times \int_0^\infty q_\rho^2 F_i^{1s_0,310}(q_\rho) F_j^{1s_0,310}(q_\rho) e^{-[(B-A)/4]q_\rho^2}\left[1 - \Phi\left(\frac{q_\rho}{2}\sqrt{A}\right)\right] dq_\rho \tag{4.6}$$

where

$$F_i^{1s_0,310}(q_\rho) = \frac{\beta^2}{96D_i^2}\left(\frac{q_\rho^4}{D_i^2} - 24\frac{q_\rho^2}{D_i} + 96\right) + \frac{\beta}{4D_i}\left(\frac{q_\rho^2}{D_i} - 8\right) + 1 \tag{4.7}$$

and similarly for the other states. In the above, we have used the notation $D_i = \beta_i + 0.7\beta$ and $A = 1/(\alpha_i + b_{Nm}) + 1/(\alpha_j + b_{Nm})$ and $B = 1/D_i + 1/D_j$. q_ρ is the ρ component of the LO-phonon wave vector and the function $\Phi(x)$ is the probability integral. $c_i^{1s_0}$ etc and I_{Z0}^{310} etc are the constants associated with the Ψ_{1s_0} etc and the Ψ_{310} etc wave functions respectively.

As can be seen from above, one advantage of using a simple expression for the metastable wave functions is that the calculation of the polaron correction is straightforward. Also, the numerators in the polaron correction terms reduce to a one-dimensional integral which can be solved readily by numerical methods.

The transition energies incorporating corrections due to the band non-parabolicity and the polaron interaction for the metastable states $|\Psi_{310}, 0\rangle$, $|\Psi_{410}, 0\rangle$ and for the hydrogenic states $|\Psi_{1s_0}, q\rangle$ and $|\Psi_{2p_{-1}}, q\rangle$ for all ranges of magnetic fields have been obtained. The results obtained are shown in figure 4 together with the new experimental data.

5. *Discussion and conclusions*

It is seen that, by modelling the excited metastable states with a very simple variational wave function, remarkably close agreement is obtained with the experimental data shown in figures 1–3. In fact, a comparison between theory and experiment on an enlarged scale clearly shows that the two agree normally to within 1% and that the maximum difference is always within 2%. Further consideration of the effects of the electron–phonon interaction has shown that even better agreement between the calculated and observed transition energies occurs when energy-dependent polaron corrections are included in the model. The form of the polaron correction given in (4.3) applies to all regions of field, provided the relevant terms are included in the sum, and it is not confined specifically to the resonant regions. However, as the simple calculation shows that the resultant correction is small, there has seemed little point in attempting such calculations for the data shown in figures 1–3. To our knowledge, this is the first time that such a good quantitative agreement has been obtained between experiment and theory for transitions involving the metastable states. It is most surprising that neglecting the coupling that exists between the Landau levels due the Coulomb term (Simola and Virtamo 1978) has so little influence on the accuracy of the calculated energies. We also note that our calculations give independent support to the original identifications of these metastable states (van Klarenbosch *et al* 1990a, b).

The polaron effect is considerably enhanced when the electronic transition energies correspond to the LO-phonon energy. The calculation of this resonant polaron interaction was described in subsection 4.2 and the results shown in figure 4. This figure shows that the splitting caused by the polaron interaction is slightly overestimated compared to the experimental values after extrapolation. The main reason for this is that many contributions to the perturbation calculations contained within (4.6) have not been included in the theory. In particular, the effects of these additional terms due to other excited states would be expected to lower the energy of the excited states involved in the transitions more than the ground state because the energy denominator is smaller, thus reducing the splitting. Hence such terms are expected to improve the agreement between theory and experiment.

Cheng *et al* (1993a) have studied the effects of confinement on the strength of the electron–LO-phonon interaction. They clearly find that the interaction is enhanced as confinement increases from three to two dimensions. This is thought to be due to the curtailment of the extent of the wave function as the well size decreases. It would be most interesting therefore to extend our current calculations on resonant polaron effects involving the excited metastable states to MQW systems.

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